International Global Navigation Satellite Systems Association
IGNSS Symposium 2020

Colombo Theatres, Kensington Campus, UNSW Australia
5 – 7 February 2020

Multi-frequency Multi-GNSS PPP: A Comparison of Two Ambiguity Resolution Methods

Viet Duong
School of Science, RMIT University, Australia
Phone: +61 4 1340 1871 Email: viet.duong@rmit.edu.au
Ken Harima
School of Science, RMIT University, Australia
Phone: +61 3 9925 3775 Email: ken.harima@rmit.edu.au
Suelynn Choy
School of Science, RMIT University, Australia
Phone: +61 3 9925 2650 Email: suelynn.choy@rmit.edu.au
Chris Rizos
School of Civil and Environmental Engineering, University of New South Wales, Australia
Phone: +61 2 9385 4205 Fax: +61 2 9385 6139 Email: c.rizos@unsw.edu.au

ABSTRACT

A major limitation of the GNSS Precise Point Positioning (PPP) technique is the slow solution convergence time. Tens of minutes are required for the solutions to converge to decimetre-level accuracy even with ambiguity resolution (PPP-AR). This paper analyses two ambiguity resolution methods to provide reliable ambiguity resolution and fast PPP convergence time. They are the partial ambiguity resolution based-LAMBDA method (PAR-Ps) and the iFlex method proposed by Trimble Navigation company. One month of multi-frequency GNSS observations from one CORS station was processed in kinematic mode. The root mean square (RMS) results indicate that the iFlex method outperforms the PAR-Ps method with respect to minimising the position errors of a kinematic test. Although two methods provide a similar horizontal convergence time (one minute), the horizontal error using the PAR-Ps is not stable until the 13-minute mark. This is caused by several sessions with incorrectly-fixed ambiguities. When compared to the PAR-Ps method, the iFlex method has improved the vertical convergence time to about 7 minutes.

KEYWORDS: Precise Point Positioning (PPP); Best Integer Equivariant (BIE); Ambiguity resolution; Convergence time; Multi-frequency and multi-constellation.

1. INTRODUCTION

Precise point positioning (PPP) has recently become a more and more popular technique in the GNSS community, as it can provide high positioning accuracy and reduced computational burden when using only a single receiver. However, the slow convergence time is the major limitation of the PPP technique. To converge to decimetre-level accuracy, even with carrier phase ambiguity resolution (PPP-AR), tens of minutes are required for the solutions (Collins and Bisnath 2011; Ge et al. 2008; Geng et al. 2010; Laurichesse et al. 2009; Laurichesse et al. 2010). The modernization of next generation Global Navigation Satellite System (GNSS) satellites, such as transmitting on three or more frequencies, is a key capability for reducing the convergence time of PPP solutions (Duong et al. 2019a; Duong et al. 2019c; Duong et al. 2016; El-Mowafy et al. 2016; Geng and Bock 2013; Laurichesse and Banville 2018; Laurichesse and Blot 2016; Li et al. 2013).

In multi-frequency and multi-GNSS applications, it is difficult to confidently fix all carrier phase ambiguities. In fact the more ambiguities, the probability of correct integer estimation (also known as the "success rate") may actually decrease (Teunissen et al. 1999). However, there may still be a subset of (linear functions of) ambiguities that can successfully be fixed. For this reason, partial ambiguity resolution (PAR) methods have been developed and can be used to fix a subset of the ambiguities.

There are several PAR approaches. Teunissen et al. (1999) introduced the first PAR method based on a minimum success rate. Another strategy is to fix only the ambiguities with the larger wavelengths, such as the (extra) wide-lane linear combinations in the case of two or more frequencies (Cao et al. 2007; Li et al. 2010). Other approaches select a subset of the ambiguities based on: (1) the standard deviation of ambiguity being below a certain level; (2) ambiguities from satellites above a certain elevation degree; (3) ambiguities with a required signal-to-noise ratio; or (4) ambiguities visible for a certain observation time (Mowlam 2004; Parkins 2011; Takasu and Yasuda 2010). In addition, another strategy fixes only ambiguities (or linear combinations of ambiguities) which are identical in the best and second-best results (Dai et al. 2007). Another consists of sequentially discarding satellites until passing a critical value in the ratio test (Duong et al. 2019b; Li and Zhang 2015; Wang and Feng 2013).

Most of the existing PAR approaches involve an iterative procedure, such as discarding satellites, which will impact processing time, which is especially critical for real-time applications or for multi-GNSS scenarios. Hence the PAR method based on success rate (Teunissen et al. 1999; Teunissen 2001a) is typically preferred. The advantage of this method is that it is easy to implement and does allow for choosing a minimum required success rate. In addition, this PAR procedure is based on decorrelation and bootstrapping and has been implemented using the Least-squares Ambiguity Decorrelation Adjustment (LAMBDA) algorithm. In this paper we will denote this method as "PAR-Ps".

However, PAR-based LAMBDA (PAR-Ps) is not able to generate reliable ambiguity solutions for multi-frequency and multi-GNSS PPP applications, especially at initial epochs (Li and Zhang 2015). In fact, the success rate is driven by the underlying model strength and not the actual measurements (Teunissen and Verhagen 2008). Any undetected biases (such as multipath effects and atmosphere biases) in the observation model may affect the data-driven success rate, although the model-driven success rate can still be high (Li and Zhang 2015). Even setting a high threshold for the success rate in the PAR-Ps method does not guarantee the correct fixing of ambiguities (Teunissen 2001b; Verhagen and Li 2012).

Several studies conducted by the Trimble Navigation company, and described in patents (Vollath and Talbot 2013), have proposed an alternative method, referred to as the iFlex method. It is claimed that the iFlex method can ensure reliable ambiguity resolution. The iFlex method is a weighted combination of some of the possible integer ambiguity candidates. In addition, Vollath and Talbot (2013) have stated that the position solution using the iFlex method will converge more rapidly to the correct values compared to the float solution. However, the performance improvement in terms of convergence time between the PAR-Ps method and the iFlex method has not been investigated. In this contribution, the positioning performance based on both the iFlex method (Vollath and Talbot 2013) and the PAR-Ps method (Teunissen et al. 1999; Teunissen 2001a) is assessed. Observations from one Continuously Operating Reference Station (CORS) for a period of 31 days in 2019 were processed in kinematic mode. These observations were then corrected using state-space representation (SSR) products, such as precise satellite orbits, clocks, code and phase biases from Centre National d'Etudes Spatiales (CNES) CLK93 real-time stream.

This paper is organised as follows. First, the GNSS observational model is presented. Next, the two PPP ambiguity resolution methods are introduced. Then, GNSS data processing strategy and selection of GNSS measurements are described. The results of the PAR-Ps and iFlex methods using multi-frequency and multi-constellation PPP-AR are analysed. The conclusions are then presented.

2. GNSS OBSERVATIONAL MODEL

The functional models of between-satellite single-differenced (SD) code and phase measurements on the i^{th} frequency at the epoch κ^{th} are given by (Hofmann-Wellenhof et al. 2008; Leick et al. 2015):

$$P_{i,(\kappa)} = \left(\rho + ct + \mu_i I_1 + T + \xi_{code,i} + \varepsilon_{code,i}\right)_{(\kappa)} \quad (1)$$

$$L_{i,(\kappa)} = \left(\rho + ct - \mu_i I_1 + T + \lambda_i \xi_{phase,i} + \lambda_i N_i + \varepsilon_{phase,i}\right)_{(\kappa)} \quad (2)$$

where P_i and L_i are the SD code and phase measurements vectors on the i^{th} frequency (in units of metres), respectively; ρ is the SD receiver-satellite geometric distance vector (m); c is the speed of light in vacuum (m/s); t is the SD satellite clock error (s); λ_i is the wavelength of the carrier phase on the i^{th} frequency (m); f_i is the i^{th} frequency (MHz); I_1 is the vector of SD first order ionospheric delays on the first frequency (m); $\mu_i = \frac{f_1^2}{f_i^2}$ is the ionosphere coefficient; T is the vector of SD tropospheric delay (m); $\xi_{code,i}$ is the vector of SD satellite code biases (m); $\lambda_i \xi_{phase,i}$ is the vector of SD satellite phase biases (m); N_i is the vector of SD integer ambiguities on the i^{th} frequency (cycles); and $\varepsilon_{code,i}$ and $\varepsilon_{phase,i}$ denotes the vector of remaining unmodelled errors such as multipath effects on the SD code and phase measurements (m), respectively.

With PPP, in order to enable high accuracy single-receiver positioning, precise satellite orbits, clocks, code and phase biases are provided to users to correct their code and phase measurements. The corrected observations at epoch κ^{th} are:

$$\bar{P}_{i,(\kappa)} = \left(P_i - ct - \xi_{code,i} = \rho + \mu_i I_1 + T + \varepsilon_{code,i}\right)_{(\kappa)} \quad (3)$$

$$\bar{L}_{i,(\kappa)} = \left(L_i - ct - \lambda_i \xi_{phase,i} = \rho - \mu_i I_1 + T + \lambda_i N_i + \varepsilon_{phase,i}\right)_{(\kappa)} \quad (4)$$

For clarity the SD receiver-satellite geometric distance vector ρ is not parameterised in terms

of receiver coordinates. Note, however, that our analyses will be based on the geometry-based model where ρ is parametrised in terms of receiver coordinates.

3. PPP AMBIGUITY RESOLUTION

The goal of geometry-based AR is to estimate the integer ambiguities, so as to improve the positioning precision. The linearised GNSS observation equations corresponding to (3) and (4) can be expressed as:

$$y = \begin{bmatrix} \bar{P} \\ \bar{L} \end{bmatrix} = Aa + Bb + \epsilon; \qquad a \in Z^n, b \in R^p \tag{5}$$

where y is the given GNSS data vector of $[m \times 1]$ which consists of the observed-minus-computed pseudorange (code) and phase observations accumulated over all observation epochs; $\bar{P}$ contains all $[\bar{P}_1^T \dots \bar{P}_i^T]^T$ and $\bar{L}$ contains all $[\bar{L}_1^T \dots \bar{L}_i^T]^T$; A $[m \times n]$ and B $[m \times p]$ are the design matrices; the entries of the vector a are the carrier phase ambiguities $[\bar{N}_1^T \dots \bar{N}_i^T]^T$, expressed in units of cycles. The entries of vector b are the remaining unknown parameters, such as receiver coordinates and atmospheric delay parameters. $\epsilon = \begin{bmatrix} \varepsilon_{code} \\ \varepsilon_{phase} \end{bmatrix}$ is the noise vector accumulated over all observation epochs; $\varepsilon_{code} = \left[\varepsilon_{code,1}^T \cdots \varepsilon_{code,f}^T\right]^T$ and $\varepsilon_{phase} = \left[\varepsilon_{phase,1}^T \cdots \varepsilon_{phase,f}^T\right]^T$ with $\varepsilon_{code,i}$ and $\varepsilon_{phase,i}$ containing, respectively, the unmodelled errors of code and phase observations on i^{th} frequency over all the epochs.

3.1 PAR-based LAMBDA

The reliability of integer ambiguity estimation depends on several factors including the strength of the underlying GNSS model and the integer estimation method that is used. As a result it is not always possible to fix all GNSS ambiguities (Teunissen et al. 1999; Verhagen et al. 2012). Even though at times all phase ambiguities cannot be reliably fixed, one could still fix a subset of the phase ambiguities (or linear functions of them) with sufficient confidence. This strategy for ambiguity resolution is referred to as partial ambiguity resolution (PAR). However, most of the PAR approaches in the literature involve an iterative procedure in which many different subsets are evaluated, which may require long search times. This limitation can be addressed when using the PAR method already proposed by Teunissen et al. (1999). The advantage of this method is that it is easy to implement and does allow for a minimum required success rate to be chosen. In addition, this PAR approach is based on decorrelation and bootstrapping, and has been implemented with the Least-squares Ambiguity Decorrelation Adjustment (LAMBDA) algorithm into a software package. A subset of the decorrelated ambiguities is fixed, with a corresponding bootstrapped success rate larger or equal to a minimum required value P_0. Hence, the goal is to select the largest possible subset such that:

$$\prod_{j=q}^{n}\left(2\Phi\left(\frac{1}{2\sigma_{\hat{z}_{j|I}}} - 1\right)\right) \geq P_0 \tag{6}$$

with $q \geq 1$. $\Phi(\mathrm{M}) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\mathrm{M}} \exp\left(-\frac{1}{2}t^2\right)dt$ is the cumulative normal distribution; $\sigma_{\hat{z}_{j|I}}$is the conditional standard deviation of the decorrelated ambiguities $\hat{z} = Z^T\hat{a}$ when using a decorrelating Z-transformation (Z); and $\hat{a}$ is the float ambiguities. Hence, only the last $n - q + 1$ entries of $\hat{z}$, denoted by $\hat{z}_s$, will be fixed. Adding more ambiguities implies multiplication with another probability, which by definition is smaller than or equal to 1. Hence, q will be chosen such that the inequality in (6) holds, while a smaller q (i.e. larger subset) would result

in a too low success rate. Note that the fixed solution in terms of the original ambiguities and the corresponding fixed remaining- unknown parameters can be obtained after applying the back-transformation as (Verhagen and Li 2012):

$$\check{a}_{PAR} = Z^{-T}\check{z}_{PAR} \tag{7}$$

$$\check{b} = \hat{b} - Q_{\hat{b}\hat{a}}Q_{\hat{a}\hat{a}}^{-1}(\hat{a} - \check{a}_{PAR}) \tag{8}$$

where $\hat{b}$ and $\check{b}$ are the float and fixed solution of the remaining unknown parameters, respectively; $\check{a}_{PAR}$ will generally not contain integer entries, since it is a linear function of the decorrelated ambiguities $\check{z}_{PAR}$, which are not all integer-valued. The corresponding precision improvement of the remaining unknown parameters can be evaluated as well with

$$Q_{\check{b}\check{b}} = Q_{\hat{b}\hat{b}} - Q_{\hat{b}\hat{a}}Q_{\hat{a}\hat{a}}^{-1}Q_{\hat{a}\hat{b}} \tag{9}$$

with $Q_{\check{b}\check{b}}$ and $Q_{\hat{a}\hat{a}}$ are the variance-covariance matrix of the unknown parameters and ambiguities, respectively; $Q_{\hat{b}\hat{a}}$ and $Q_{\hat{a}\hat{b}}$ are the submatrices.

3.2 The iFlex method

The iFlex method is described by Vollath and Talbot (2013). This method is a weighted combination of some of the possible integer ambiguity candidates. The iFlex method has been found to converge more rapidly to the correct values in a reasonable computation time as compared with the convergence time of the float solution. This method relies on the best integer equivariant (BIE) method when the observations (y) follow a Gaussian (or normal) distribution. In that case, the PDF of y takes the form:

$$p_y(y) = \frac{1}{(2\pi)^{\frac{m}{2}}\sqrt{detQ_y}} \exp\left\{-\frac{1}{2}\|y - Aa - Bb\|_{Q_y}^2\right\} \tag{10}$$

where $\|\cdot\|_{Q_y}^2 = (\cdot)^T Q_y^{-1}(\cdot)$, and det denotes the determinant operator. As shown by Teunissen (2003), in the case the vector of observations y follows a Gaussian distribution, the BIE estimators of ambiguities a and the remaining real-valued parameters b will be simplified to:

$$\begin{cases} \hat{a}_{BIE} = \dfrac{\sum_{z\in Z^n} z\, exp\left(-\dfrac{1}{2}\|\hat{a} - z\|_{Q_{\hat{a}}}^2\right)}{\sum_{z\in Z^n} exp\left(-\dfrac{1}{2}\|\hat{a} - z\|_{Q_{\hat{a}}}^2\right)} = \displaystyle\sum_{z\in Z^n} z w_z(\hat{a}) \\ \hat{b}_{BIE} = \hat{b} - Q_{\hat{b}\hat{a}}Q_{\hat{a}}^{-1}(\hat{a} - \hat{a}_{BIE}) \end{cases} \tag{11}$$

with $\sum_{z\in Z^n} w_z(\hat{a}) = 1$ and weighting factor $w_{z_j}(\hat{a}) = \frac{exp\left(-\frac{1}{2}\|\hat{a}-z_j\|_{Q_{\hat{a}}}^2\right)}{\sum_{z\in Z^n} exp\left(-\frac{1}{2}\|\hat{a}-z\|_{Q_{\hat{a}}}^2\right)} \leq 1, \forall z \in Z^n$. Note that the weighting factor of each integer candidate will be influenced by choosing distribution of the observations (y), and thus the squared norms of integer candidates $\left(\|\hat{a} - z\|_{Q_{\hat{a}}}^2\right)$. Vollath and Talbot (2013) also present empirical formulas, namely the Laplace or a Minmax distribution, which may be used to calculate the weighting factor of (11) according to the following expressions:

$$w_{z_j}(\hat{a}) = \frac{exp\left(-\alpha\sqrt{\|\hat{a}-z_j\|_{Q_{\hat{a}}}^2}\right)}{\sum_{z\in Z^n} exp\left(-\alpha\sqrt{\|\hat{a}-z\|_{Q_{\hat{a}}}^2}\right)} \ (Laplace) \tag{12}$$

$$w_{z_j}(\hat{a}) = \frac{exp(-max|\hat{a}-z_j|)}{\sum_{z\in Z^n} exp(-max|\hat{a}-z|)} (Minmax) \tag{13}$$

where α is a scaling factor that can be used to tune the weights and $|\cdot|$ is the absolute value.

The BIE estimation $\hat{a}_{BIE}$, and thus $\hat{b}_{BIE}$, in (11) cannot be computed exactly because of the infinite sum over integers. If the infinite sum is replaced by a sum over a finite set of integers, this might result in an estimator that is not integer equivariant anymore. In GNSS applications it is necessary to find an approximate solution for this estimator while retaining the property of integer equivariance. Hence, when implementing (11) in any software package, the size of the integer sets must be set beforehand to reduce the computational burden. Vollath and Talbot (2013) state that when forming the weighted average of the iFlex method, selecting too many candidate sets does not substantially improve the convergence of the iFlex method, but will increase the computational burden. To tackle this problem, they proposed a method to limit the size of the integer candidate sets. The basis of this approximation method is to apply an empirical threshold (i.e. critical value) to only include candidate sets having a significant weight with respect to the best (first) integer candidate (the one with the shortest squared-norm distance), such that:

$$\frac{w_{z_j}(\hat{a})}{w_{z_1}(\hat{a})} > \gamma \tag{14}$$

where $w_{z_1}(\hat{a})$ is the weight of the first candidate set of ambiguities (z_1); $w_{z_j}(\hat{a})$ represents the weight of the j^{th}candidate set; and γ is an empirical value and set to a value of 0.01 or 0.001 (Banville 2016; Vollath and Talbot 2013). This means that the discarded integer candidate vectors have a weight of at least 100 or 1000 times smaller than the first integer candidate set.

4. DATA COLLECTION AND PROCESSING STRATEGY

To evaluate the PPP-AR performance using two different ambiguity resolution methods, real GNSS measurements were collected from one GNSS station, namely ALBY, over 31 consecutive days from April 10 to May 10, 2019 (DOY 100 to 130). The reference station must track three frequencies (GPS block IIF and BeiDou) and four frequencies (including Galileo E6 signal) simultaneously. An average of six one-hour sessions in a day were selected on the basis that the measurements contain the maximum number of triple- and quad-frequency satellites in view for all three constellations. A total 180 datasets were analysed. Due to large system biases of the BeiDou GEO satellites, only the BeiDou IGSO and MEO satellites were used in this study.

It should, however, be pointed out that the number of GNSS satellites transmitting three (GPS satellites from Block IIF or BeiDou constellations) or four frequencies (Galileo satellites) was limited during certain periods of the day. Therefore, the dual-frequency GPS satellites were also utilised to provide a better geometric information. In our work, all available frequencies from three GNSS constellations were used. The triple-frequency (GPS+BeiDou) and quad-frequency (Galileo) PPP solutions were based on a combination of dual- and triple-frequency GPS measurements, triple-frequency BeiDou measurements along with quad-frequency Galileo measurements.

A modified version of the RTKLIB software (Takasu 2013) was used to stream the GNSS measurements and output the corrected observables free from the systematic effects that must be considered for PPP processing (i.e. phase windup). A MATLAB-based GNSS PPP software

with a Kalman filter estimator was developed to process the corrected observables using the multi-frequency uncombined observation model (or UC) for GPS, Galileo and BeiDou. Note that henceforth the phrase "uncombined observation models" implies no ionosphere-free linear combination of measurements were created and that the ionospheric delay was estimated along with the other parameters using the single-differenced measurements.

The commonly used variance function of phase observations, dependent on the satellite elevation angle θ, is:

$$\sigma_L^2 = \frac{\sigma_{L_0}^2}{sin^2\,\theta} \tag{15}$$

where the standard deviation of GPS and Galileo phase observations at zenith direction $\left(\sigma_{L_0}\right)$ was chosen to have the value of 3mm. The measurement error ratio between code and carrier phase observations $\left(\frac{\sigma_P}{\sigma_L}\right)$ was set to be 100, except for BeiDou where the value of 200 was adopted. Considering the stochastic information of the real-time satellite orbit, clock and bias products in the stochastic model, the formula (15) now has been modified as:

$$\bar{\sigma}_L^2 = \left(\frac{\sigma_{L_0}^2}{sin^2\,\theta}\right) + \sigma_{clk}^2 + \sigma_{orb}^2 + \sigma_{bias}^2 \tag{16}$$

where σ_{clk}^2 is the satellite clock uncertainty, σ_{orb}^2 is the orbit uncertainty, and σ_{bias}^2 is the uncertainty of the satellite phase and code biases.

Table 1 Summary of multi-frequency multi-GNSS PPP-AR processing strategy

Items	Models/Constraints
Station coordinates	Estimated in PPP in kinematic mode
Observations	-GPS (L1, L2, L5)/GAL (E1, E5a, E5b, E6)/BDS (B1, B2, B3) -Elevation-dependent weighting strategy Eq. (16)
Elevation cut-off angle	10º
Sampling rate	1 s
Precise satellite orbit, clock, code (considering code elevation-dependent for BDS) and phase biases	CNES CLK93 real-time stream
Satellite and receiver phase centre offset (PCO) and phase centre variation (PCV)	Corrected using IGS antenna products (igs14_2013.atx)
Phase wind-up	Corrected
Slant ionosphere	Estimated as parameter (random-walk process with a constraint of 0.002m/sqrt(s))
Troposphere model	Zenith hydrostatic delay is obtained by Saastamoinen model using a standard atmospheric model. Zenith wet delay is estimated as a random-walk parameter with process noise 5e-6m/sqrt(s). Global Mapping Function (GMF) used to map the slant tropospheric delay to zenith
Displacement	Solid earth tides, pole tides, ocean tide loading correction (FES2004) and relativistic effects modelled using the IERS Conventions 2010
Ambiguities	Estimated as constant

5. PPP RESULTS

To evaluate the performance of different ambiguity resolution methods on PPP position convergence, the triple-frequency (GPS+BeiDou) and quad-frequency (Galileo) measurements were processed using two different ambiguity resolution approaches: (1) the PAR-Ps method, and (2) the iFlex method using two different weighting functions in (12) and (13), referred to as "iFlex-Laplace" and "iFlex-Minmax", respectively.

Table 2 Threshold values for ambiguity resolution methods

Methods	**Critical values**
PAR-Ps	Success rate: $P_0 \geq 0.999$
iFlex method with weighting functions based on (12) and (13)	$\gamma = 0.001$ and $\alpha = 0.25$

Figure 1 shows the horizontal and vertical root mean square (RMS) as a function of time for the three solutions calculated with two different ambiguity estimators, namely the iFlex method and the PAR-Ps method. Note that two different weighting functions (Laplace and Minmax) of the iFlex method were also included in the comparison. In addition, the RMS is computed from the differences between the "known" coordinates of the ALBY station with the position estimates provided by the PPP-AR solutions. The average RMS curves highlight the benefit of iFlex method which converges to 0.1m horizontal components and 0.2m vertical component in about 1 and 6 minutes, respectively. The PAR-Ps method requires 1 and 13 minutes to reach the same level of accuracy. Although two methods provide a similar horizontal convergence time (one minute), the horizontal error in the PAR-Ps case is not stable until the 13-minute mark. This is caused by several sessions with incorrectly-fixed ambiguities, resulting in a large position error. The iFlex method has a slight advantage over the PAR-Ps method in the case of the vertical component. When compared to the PAR-Ps method, the iFlex method has an improved vertical convergence time of about 7 minutes.

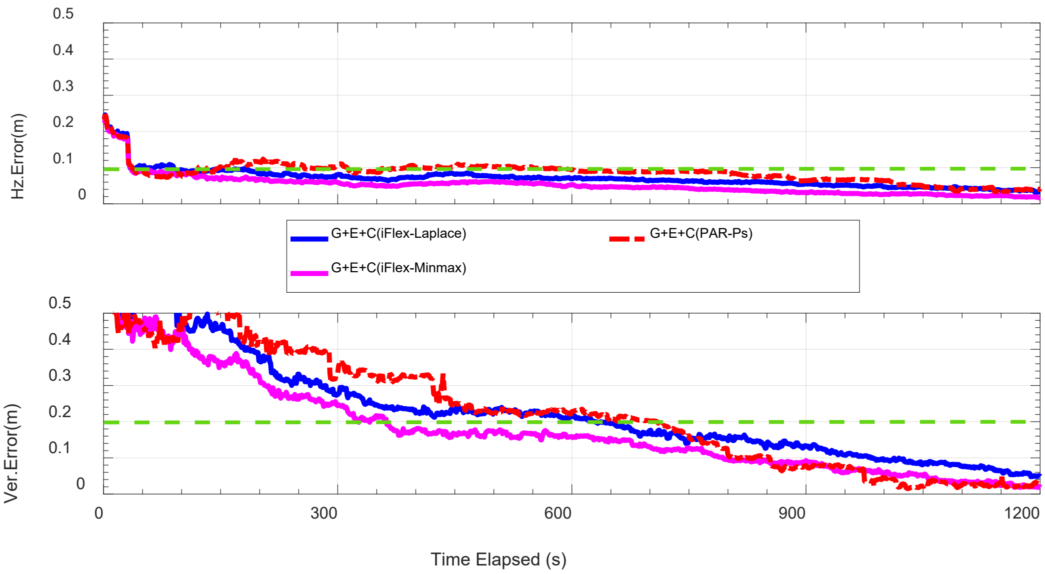

Figure. 1. Time series of the horizontal (top) and vertical (bottom) RMS errors based on the combined GPS+Galileo+BeiDou (G+E+C) measurements used for two different PPP-AR methods, namely the iFlex method and the PAR-based LAMBDA method (PAR-Ps). The blue and magenta solid lines represent the iFlex method using Laplace and Minmax, respectively. The dashed red line displays the solution of the PAR-Ps method. These combined RMS values are based on the PPP-AR results from all consecutive days.

It is worth mentioning that the computation time using the iFlex methods is 3 to 4 times longer compared to using the PAR-Ps method. The reason is that the calculated results of the iFlex methods are based recursive calculations, by taking integer candidates for the summations, as shown in (12) and (13). As a result, the iFlex method typically takes a longer computation time compared to the PAR-Ps method.

6. CONCLUDING REMARKS

Modernisation of GNSS satellites offers additional signal frequencies, which allows for lower PPP solution convergence time. In multi-frequency and multi-GNSS scenarios there is a need to obtain an accurate and reliable estimate of the receiver position. In this research, we have assessed two different carrier phase ambiguity resolution methods for multi-frequency, multi-GNSS PPP, namely the PAR-based LAMBDA method (PAR-Ps) and the iFlex method. Analysis of real triple- and quad-frequency GNSS measurements has shown that the iFlex method outperformed the PAR-Ps method in the sense of minimising the position errors of a kinematic test. Although two methods provide a similar horizontal convergence time (one minute), the horizontal curve in the PAR-Ps method is not stable until the 13-minute mark. This is caused by several sessions with incorrectly-fixed ambiguities, resulting in a large position error. When compared to the PAR-Ps method, the iFlex method has improved the vertical convergence time to about 7 minutes. It is anticipated that with an improved accuracy of the Galileo and BeiDou satellite correction products, provision of precise atmospheric delay corrections, as well as an increased number of multi-frequency GNSS satellites, and better geometry in the near future, instantaneous PPP-AR may be a reality.

ACKNOWLEDGEMENTS

Geoscience Australia and the IGS are acknowledged for providing the GNSS data as well as the satellite corrections. The Raijin-NCI National Computational Infrastructure Australia is acknowledged for providing high-performance research computing resources for GNSS data processing. The first author is supported by the Australia Award Scholarship Scheme to pursue a Ph.D. at RMIT University, Melbourne, Australia.

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